office hour week 9(a)

LAST Name: ANSWER.

First Name: _____

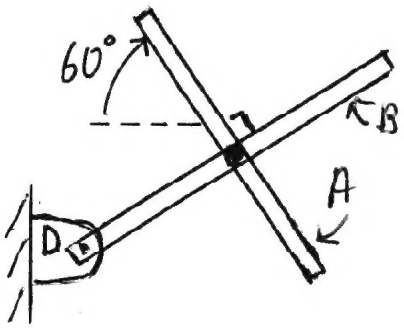
Student Number: _____

MIE 200F - Quiz number 6b - November 15/00
quiz duration = 25 minutes

Two rods each have a mass of 4 kg and a length of 6 meters. They are welded together at their centers as shown The structure is released at the position shown, and allowed to accelerate in a clockwise direction under the force of gravity.

(a) Find the moment of inertia of the structure about point "D"

(b) Find the angular acceleration at the position shown in the diagram.



$g = 9.81\ m/s^2 \downarrow$ $\quad T_2 = T_1 + U_{1\to 2}$

$I_{c\ of\ g}$ for a rod $= m L^2/12$ $\quad I_o = I_{c\ of\ g} + m d^2$

$I_{c\ of\ g}$ for a uniform disk $= m R^2/2$

$V_P = V_Q + V_{P/Q}$ $\quad |V_{P/Q}| = \omega R_{PQ}$

$a_{P/Q} = -(\omega^2 R_{PQ})\hat{e}_r + (\alpha r)\hat{e}_\theta$ $\quad a_P = a_Q + a_{P/Q}$

$T = \frac{1}{2} m (v_{c\ of\ g})^2 + \frac{1}{2} I_{c\ of\ g} \omega^2 = \frac{1}{2} I_o \omega^2$

(a)

$$I_D = (I_A)_D + (I_B)_D$$

$$= \left(\tfrac{1}{12} m_A L_A^2 + m_A d^2\right)_D + \left(\tfrac{1}{12} m_B L_B^2 + m_B d^2\right)_D$$

$$= \left(\left(\tfrac{1}{12}\right)(4)(6^2) + (4)(3^2)\right) + \left(\left(\tfrac{1}{12}\right)(4)(6)^2 + (4)(3)^2\right)$$

$$I_D = (12 + 36) + (12 + 36) = 96\ kg\,m^2$$

(b) $\quad \Sigma M_D = I_D \alpha$

$+\circlearrowleft$ $\quad (4kg + 4kg)(9.81)(3\sin 60) = 96\alpha$

$$203.9 = 96\alpha$$

$$\alpha = 2.12\ s^{-2} \circlearrowleft$$

You must include units & direction

week 9(b)

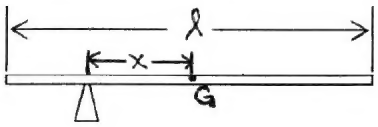
Answers

MIE 200F – Quiz 6a – November 12, 2003
Quiz Duration: 20 min

A uniform slender rod of mass 2 kg and length 3 m is released from rest in the position shown. The point G indicates the centre of mass of the rod which is located at the rod's midpoint.
Calculate:

(a) Find the angular acceleration, α, for the rod as a function of x
(b) For what value of x is α a maximum?

$\Sigma F = m\bar{a}$ $\quad T_2 = T_1 + U_{1\to 2}$
For a thin rod: $I_G = \frac{1}{12}ml^2$
$I_o = I_G + md^2$ $\quad T = \frac{1}{2} I_o \omega^2$



Solution:

(a) $\Sigma M_o = I_o \alpha$

$$mgx = \left(\frac{1}{12}ml^2 + mx^2\right)\alpha$$

$$\alpha = \frac{gx}{\frac{l^2}{12} + x^2}$$

$$\boxed{\alpha(x) = \frac{gx}{\frac{3}{4} + x^2}}$$

(b) α is a maximum when $\frac{d\alpha}{dx} = 0$

$$\frac{d\alpha}{dx} = \frac{\left(\frac{3}{4} + x^2\right)(g) - gx(2x)}{\left(\frac{3}{4} + x^2\right)^2} = 0$$

$$\frac{3}{4} + x^2 - 2x^2 = 0$$

$$x^2 = 3/4$$

$$\boxed{x = \frac{\sqrt{3}}{2}}$$

week 9(c)

ANSWERS.

LAST Name: _____

First Name: _____

Student Number: _____

MIE 200F – Quiz 6B – November 13, 2003

Quiz Duration: 25 minutes

A solid homogeneous cylinder of radius 200mm and mass of 20 kg is placed on a 25° incline. A steady force P (50N) is applied to the center of mass G at an angle of 60° from the horizontal as shown in the figure. The coefficients of static and kinetic friction are $\mu_s = 0.15$ and $\mu_k = 0.08$. Determine:

a. The acceleration of the mass center G.

b. The friction force exerted by the ramp on the cylinder.

Equations: $I_{C\ of\ G} = \frac{1}{2}mr^2$ $\sum \vec{M} = I\vec{\alpha}$ $\sum \vec{F} = m\vec{a}$

Solution:

$\sum F_x = ma_x$

$mg\sin 25 - P\cos 35 - f = ma$

$(20\times 9.81\times \sin 25) - (50\times \cos 35) - f = 20a$ (1)

$\sum F_y = ma_y$

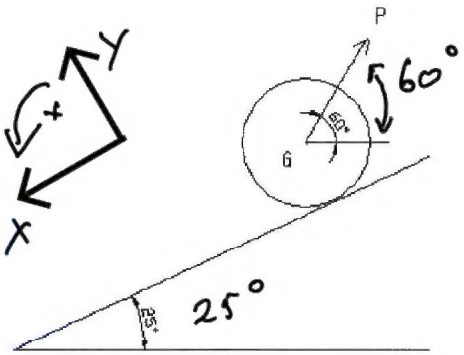
$-mg\cos 25 + P\sin 35 + N = 0$

$-(20\times 9.81\times \cos 25) + (50\times \sin 35) + N = 0$ (2)

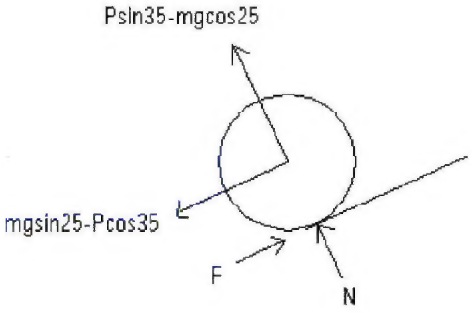
$\sum M_G = I_G\alpha$

$rf = \frac{1}{2}mr^2\alpha$

$f = \frac{1}{2}mr\alpha$ (3)



Assume rolling with no slip: $a = r\alpha$ $\therefore f = \frac{1}{2}ma$ (4)



$|a| = 1.399 m/s^2$

$|f| = 13.99$ Newtons ; $|N| = 157$ Newtons

$f_{max} = \mu_s N = 22.37$ Newtons $> f$

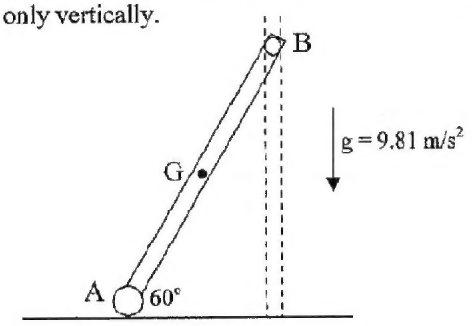
Therefore assumption is valid

office hr – week 9(d)

Pre-Exam review: April 2013 MIE100S - Dynamics A.N. Sinclair

1. The long thin stick AB has a mass m = 4 kg and a length 7 meters. It is released from rest at the position shown. Assume that friction and air resistance are negligible. Point "A" is constrained so that it can travel only horizontally; Point "B" is constrained so that it can travel only vertically.

(a) Write down the equations needed to find the angular acceleration α at the position shown.
(b) At what speed does point "B" hit the floor?



SOLUTION TECHNIQUE & STEPS

(a) We identify that this is a 2-D kinetics problem, where we are trying to determine an acceleration component at a given instant in time. Our basic tools for such problems are:

(i) $\sum \vec{F} = \vec{a}_G$; (ii) $\sum M_o = I_o\alpha$; (iii) $\sum M_G = I_G\alpha$

We note that equation (i) is a vector equation, so it is really two equations combined into one. It can be broken down into x and y, or n and t, or r and θ components.

In general, you can use either equation (ii) or (iii) but not both. Equation (ii) leads to easier arithmetic in the solution, but it requires a fixed axis. There is no fixed axis for this problem, so we are left with equations (i) and (iii), plus the kinematic relations for relative acceleration and velocity.

Choose your axes. Your best bet is x (horizontal) and y (vertical), as this means that the accelerations of points A and B will each line up nicely with one of the axes. This will make the arithmetic simpler when applying equation (i). Also select a direction for positive rotation. Stick with those choices for all variables and equations.

Draw a free body diagram. If you are uncertain about the direction of a particular force, then assume that it acts in the positive direction. (If this assumption turns out to be wrong, then you'll get a negative sign when you solve for that particular force, and so you'll know it actually acts in the negative direction).

Apply the kinetic equations (i) & (iii). Decompose all forces, and $\vec{a}_G$, and equation (i) into x and y components.

If you have more unknowns than equations, then eliminate some unknowns by using the kinematic relationships. Pay careful attention to your positive/negative directions of axes and rotation when composing this new equation(s). You should now have enough equations to solve for all unknowns. (see next page)

(b) This is a "journey" or "displacement" type of problem. This means that you are given a starting configuration (labeled position 1), and then the body is exposed to some forces that displace or rotate it to a new position (labeled position 2); you are then asked to find the final velocity or distance traveled at position 2.

To solve a "journey" type of problem, you can try to use equations (i), (ii), and (iii) above to find $\vec{a}_G$ and α, and then integrate those variable once to find the final velocity, or integrate twice to get the final position. This can be really difficult if $\vec{a}_G$ and α vary in a complicated way during the journey, as they do in this case.

Alternatively, for journey problems such as this, we can try to use the work-energy equation provided that we know how to calculate the work done, and provided we haven't lost an unknown amount of energy through generation of heat, sound, air resistance, etc. These favorable conditions are present for this problem.

(If there are interactions among multiple particles, collisions, or explosions in a journey problem, then the translational momentum-impulse equation is useful, but that isn't the case here.)

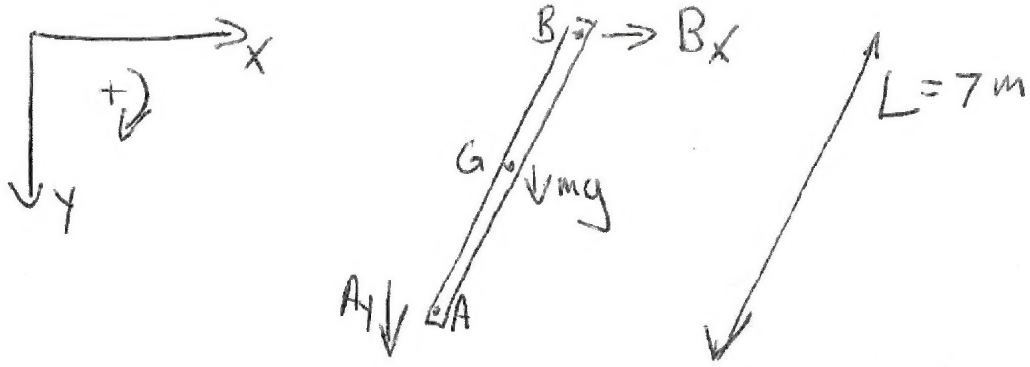
Examine carefully the geometry of the falling stick. As point "B" is about to hit the ground, the velocity of "A" goes to zero. This means that it becomes the instantaneous center of zero velocity. At that final instant, $T = \frac{1}{2} I_A \omega_2^2$

Only gravity does work. $T_2 = T_1 + \text{work}_{\text{gravity}} \Rightarrow \frac{1}{2} I_A \omega_2^2 = 0 + (-mg\Delta h) \Rightarrow$

$$\frac{1}{2}\left[\left(\frac{1}{12}\right)(4kg)7^2 + (4kg)\left(\frac{7m}{2}\right)^2\right]\omega_2^2 = 0 - (4\,kg)(9.81)\left(\frac{-7m}{2}\sin 60\right) \Rightarrow \omega_2 = 1.91\ s^{-1}$$

Because "A" is instantaneous center of zero velocity at end of journey, final $v_B = \omega_2 L = (1.91)(7) = 13.4\ m/s$

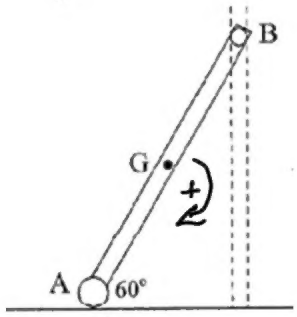
1 (a)



① $\Sigma F_Y = m(a_G)_Y \Rightarrow mg + A_Y = m(a_G)_Y$

② $\Sigma F_X = m(a_G)_X \Rightarrow B_X = m(a_G)_X$

③ $\Sigma M_G = I_G \alpha \Rightarrow (B_X)(\frac{L}{2}\sin 60^\circ) - (A_Y)(\frac{L}{2}\cos 60^\circ) = (\frac{1}{12} m L^2)\alpha$



Unknowns $(a_G)_X$, $(a_G)_Y$, α, A_Y, B_X

Kinematic: $\vec{a}_G = \vec{a}_A + \vec{a}_{G/A}$

$$\left[\vec{a}_G = \vec{a}_A + \omega^2 r_{AG}\,\hat{u}_n + \alpha r_{AG}\,\hat{u}_t\right]_Y$$

④ $\Rightarrow (a_G)_Y = 0 + 0 + \alpha \frac{L}{2}\cos 60^\circ$ (60° from Y → u_t)

$$\vec{a}_G = \vec{a}_B + \vec{a}_{G/B}$$

$$\left[\vec{a}_G = \vec{a}_B + \omega^2 r_{GB}\,\hat{u}_n + \alpha r_{GB}\,\hat{u}_t\right]_X$$

⑤ $(a_G)_X = 0 + 0 + \alpha \frac{L}{2}\cos 15^\circ$ (u_t → X)